A mathematical model for interpretation of antibody titre distribution

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Context and objective

Measurements of individual antibody titres define of a distribution of antibody titres at the herd level (e.g. Fig.1 with 3 antibody titre levels).

This distribution can potentially contain information on time since virus introduction

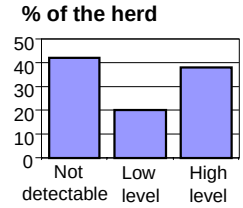


Fig 1. Example of antibody titre distribution

Objective :

To define a modelling framework to interpret distribution of antibody titres at herd level in terms of possible period of time since pathogen introduction.

Mathematical model

- Stochastic SIR model with *K* antibody titre levels (*K=3* in Fig.2).
- Dynamic of the antibody titre:
  - Increase after an infection (I) or a re-exposure (R),
  - Decrease if maternal antibodies (S) and after the infection (R).
- Horizontal transmission (rate $\lambda(S_\alpha \rightarrow I_\alpha)$) : frequency-dependent form, taking into account a variability of the sensibility (q_α) and of the excretion (p_α) related to the antibody titre level α.

$$\lambda(S_\alpha \rightarrow I_\alpha) = q_\alpha c\left(p_1\frac{I_1}{N} + p_2\frac{I_2}{N} + p_3\frac{I_3}{N}\right) \quad \begin{cases} 0 \le p_3 \le p_2 \le p_1 \le 1 \\ 0 \le q_3 \le q_2 \le q_1 \le 1 \end{cases}$$

where I_α is the number of infected animal with the antibody titre level α and *N* the total number of animals.

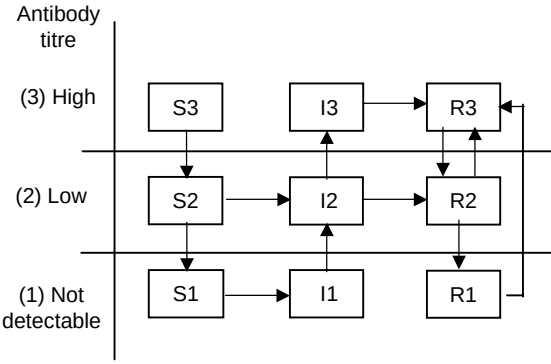


Fig 2. Diagrammatic representation of the model: $S\alpha \leftrightarrow$ susceptible with the antibody titre level α, $I\alpha \leftrightarrow$ infected with the antibody titre level α, $R\alpha \leftrightarrow$ recovered with the antibody titre level α; α∈{1 (not detectable), 2 (low), 3 (high)}.

Model use

1. Definition of the model for the considered pathogen and the population
2. Simulation of the model
3. For a given distribution (e.g. Fig.1), definition over time of the probability to have the observed distribution (e.g. Fig. 3 with 5000 simulations)

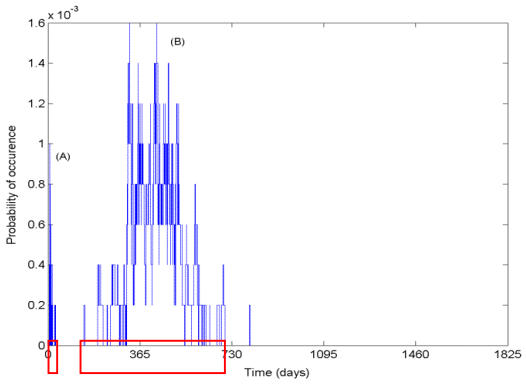
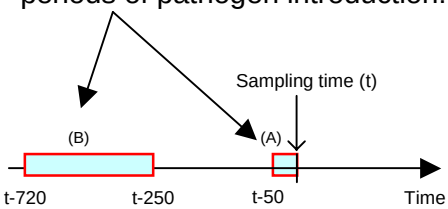


Fig 3. Proportion of simulations, over time, where the considered distribution was observed

4. Identification of most likely periods of pathogen introduction:



Discussion and conclusion

- Uses the dynamics of antibody titre distribution.
- Sensitivity and specificity of the test need to be considered.
- Other possible use: Determination of the probability that the pathogen was present at the time of sampling.