Applications of stochastic games to veterinary epidemiology

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Introduction

Farmer behaviour is an important factor in determining rates of disease transmission. How farmers manage disease through monitoring for its presence and controlling its spread has the potential to change the dynamics of disease spread. Markov chain analysis was first introduced into epidemiology by Gani and Jerwood [3]. who pointed out for the first time that so-called chain binomial models (originally due to En'ko) could in fact be characterised by Markov chains. Around the same time epidemiologists began applying stochastic dynamic programming to problems of disease control [1]. Stochastic games of disease spread extend this line of research to the case of multiple interacting decision makers. Stochastic games [4] allow us to incorporate both disease dynamics using a Markov chain model of disease transmission and behavioural feedbacks in a natural way. Game payoffs are are based on farm profitability and data for these are readily obtainable from publicly available data. Transition probabilities depend on farmer decisions and disease dynamics and parameters for these may be obtained from epidemiological studies.

Agricultural dilemmas

Agricultural dilemmas have a long history. A version of the prisoner's dilemma (the farmer's dilemma) was proposed by Hobbes and elaborated on by Hume. We consider a veterinary version of social dilemmas that may arise in different states of nature corresponding to different disease states. Table 1 contains the payoff to each farmer from different action combinations. The first number in each cell is the payoff to the row players and the second the payoff to the column player. A Nash equilibrium arises when neither player has incentive to change their strategy. The Nash equilibrium is highlighted in colour.

Table 1 : Veterinary dilemma

	Treat	Don't Treat
Treat	7, 7	1,10
Don't Treat	10,1	3,3

By not treating a farmer seeks to free-ride on the other farmer and benefit from herd immunity. However if neither treat there is a risk of disease spreading. Spread of disease is dynamic and cannot be captured in a static model like that presented above so we introduce disease dynamics using Markov chains.

By introducing dynamics we are able to obtain a richer set of equilibria compared with the static case and uncover behaviour that we would not observe in a static game. In a stochastic game payoffs are state dependent and depend on the underlying disease state (see Table 2 and 3). Transition probabilities from one disease state to another depend on the joint decisions of all farmers.

Stochastic games

An n-player stochastic game involves maximizing the discounted expected returns for each player by choosing actions subject to the state of game evolving according to a Markov chain. Equilibria are mixed strategy Nash equilibria. These are found by solving an equivalent non-linear programming problem. Stochastic games extend stochastic dynamic programming to situations with multiple decision-makers whose decisions impact each others payoffs. They are a very general class of game theoretic models that allow both discrete and continuous-states and decision and can be extended to incorporate incomplete information and information revelation such as that obtained through diagnostic tests. Stochastic games are well suited to studying bioeconomic problems with multiple decision-makers. However, they have not previously been applied to epidemiology.

Example: Endemic equilibrium trap

In tables 2 and 3 the numbers in brackets below the diagonal are transition probabilities. The first number is the probability of transitioning to state 1 and the second the probability of transitioning to state 2.

There are three equilibria. Two pure strategy and a mixed strategy equilibria in state 1 and a single pure strategy equilibria in state 2. The pure strategy equilibrium outcomes are highlighted in colour. Farmer decisions in conjunction with these transition probabilities determine whether the state is able to cycle or not. In our case we obtain the result that cycling only occurs if the farmers are sufficiently patient.

Table 2 : Disease present (state 1)

	Treat	Don't Treat
Treat	7, 7 $(0,1)$	1,10 $(0.5,0.5)$
Don't Treat	10,1 $(0.5,0.5)$	3,3 $(1,0)$

Table 3 : Disease free (state 2)

	Treat	Don't Treat
Treat	8,8 $(0,1)$	8,12 $(0.5,0.5)$
Don't Treat	12,8 $(0.5,0.5)$	12,12 $(1,0)$

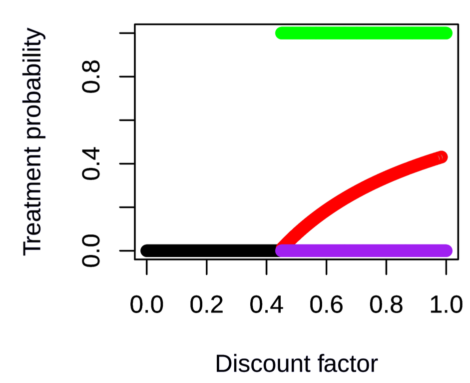

Figure 1 : Probability of treatment in state 1

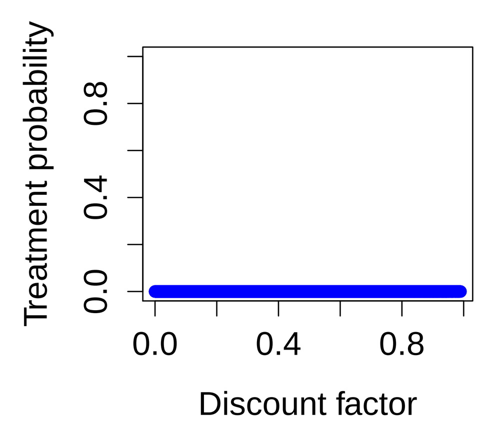

Figure 2 : Probability of treatment in state 2

A low discount factor corresponds to farmers valuing the present much more than the future (myopic behaviour). Figure 1 shows how increased time discounting (a lower discount factor) reduces the probability of treating in state 1 (disease present). The state therefore is unable to switch to the disease free state when players are myopic. Figure 2 shows that if one were already in the disease free state then in equilibrium one would never treat. The state then transitions to state 1 (disease present) leading to an endemic equilibrium trap. If farmers are sufficiently patient the mixed strategy equilibrium leads to epidemic cycles.

Conclusion

Disease dynamics and economic considerations may be usefully analysed using stochastic games as a means of analysing new veterinary measures such as diagnostic tests [2]. We are extending the approach to the study of an endemic disease: sheep-scab. This will allow us to account for for disease dynamics based on a compartmental model in which transition between states is jointly determined by farmer behaviour. In the example presented here we say how farmer time preferences may other things equal, impact whether a disease becomes endemic or enters a series of epidemic cycles.

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