Comparison of horizontal transmission functions: A theoretical model of a pathogen spread

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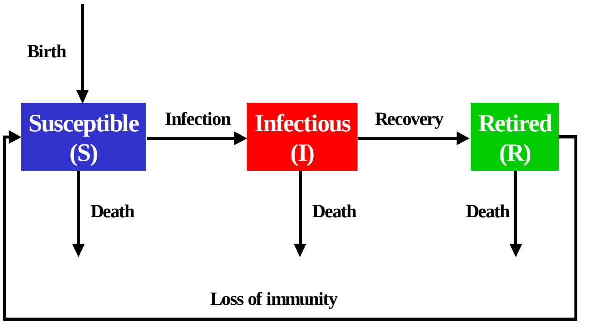
INTRODUCTION

Horizontal transmission in a herd is the process by which susceptible animals become infected by contact with the infectious animals. A mathematical formalization, through a so-called transmission function, is used to represent this process and is incorporated in epidemiological models. The transmission function is recognized as a major driving force of these global models.
Here we present a comparison of different transmission functions encountered in the literature, through their influence in a simple theoretical model of a pathogen spread.

MODEL DESCRIPTION

Model structure

The model is of classical *SIR* (Susceptible-Infectious-Retired) type
Two situations are considered:
1- the size of the population (*N*) is constant: death rate is equal to birth rate and does not depend on infection. Both rates are a proportion of the population size.
2- the size of the population is variable: "natural" death rate is lower than birth rate but a specific death rate is due to the infection.
In both cases, the area occupied by the population is constant, so that considering variables as numbers (as in this case) or densities is equivalent.



Equations of the transmission functions

The transmission function represents the number of animals *S* that become *I* per unit of time. Five different types of transmission functions were used in the model:

1- Density-Dependent (**DD**), this type of function is usually associated with an homogeneous random mixing. Equation is as follows: βSI

2- Frequency-Dependent (**FD**), often applied to heterogeneous contact structures, with the following equation: $\beta \frac{SI}{N}$

3- "Refuge" effect (**R**), derived from density-dependence with local heterogeneities, through aggregation parameter q: $\beta I \left(N - \frac{I}{q} \right)$

4- Negative Binomial (**NB**), where non linearity accounts for an heterogeneity of risk of susceptible hosts (k: index of aggregation): $kS \ln\left(1 + \frac{\beta I}{k} \right)$

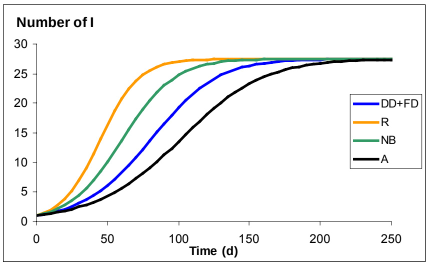
5- Asymptotic (**A**), which corresponds to a frequency-dependent function with a saturation (saturation constant c): $\beta \frac{SI}{c+S+I}$

In order to compare results simulated with these transmission functions, the same R_0 (basic reproductive ratio) was considered for the different models. R_0 was estimated with a constant population size by the ratio N/S*, with S* representing the number of Susceptible at equilibrium. R_0 value gives conditions for the value of β (transmission coefficient).

RESULTS

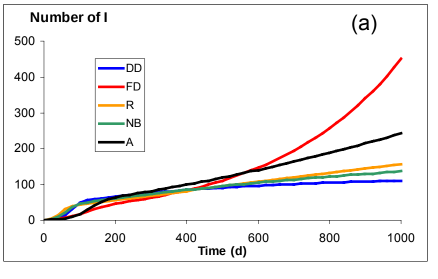
Constant size of population (N=100)

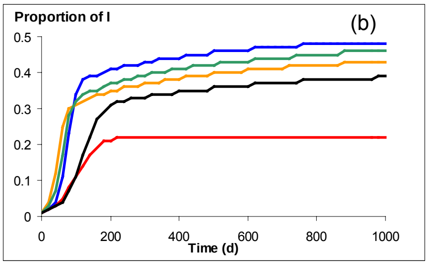
The number of Infectious individuals tends to a plateau (here 27.5) for any function considered. By construction, DD and FD are confounded. The time at which the plateau is reached differs between functions. Models with functions DD or FD are intermediate between models with R or NB which reach this value earlier and with A, for which the increase of *I* is slower.



Variable size of population

Differences between transmission functions can more easily be noticed in a case when the size of the population may increase with time. More particularly, the evolution of the number of infectious animals (a) underlines a distinction between a continuously growing population of infectious (functions FD and A) and a case where a plateau is reached (DD, R and NB). Conversely, whatever the function used, the proportion of infectious tends to a maximal value (b). However, this value differs between functions, with DD and FD on the opposite from each other, and the time to reach this value also differs between the different models. The high size of the population with FD and its continuous increasing explains that, in this case, the proportion of infectious is low although their number increases in an exponential way.





CONCLUSIONS

This simple modelling approach underlines the need for a careful choice of the transmission function to be included in an epidemiological model. The simulations show differences in the results obtained with the different functions, with a distinction between density-dependence and related functions (DD, R and NB), and frequency-dependence and derived asymptotic function (FD and A). Moreover the behaviour of the models including the different functions can be mostly retrieved by an analytical study of the equations. The choice of an adequate transmission function has mainly to be supported by a biological grounding. The distinction between density- and frequency-dependent functions is often related to the difference between homogeneous and heterogeneous contact structures. In a case of a constant size herd, no clear distinction between these main functions can be made.